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PARAMETRIC STUDY WITH GEOFRAC: A THREE-DIMENSIONAL STOCHASTIC FRACTURE FLOW MODEL

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ABSTRACT

This paper presents results of a parametric study using GEOFRAC. Recent developments in GEOFRAC allow the user to calculate the flow in a fractured medium in a geothermal context. Both the fracture- and the flow model have been tested, and a simple parametric study was conducted in order to check the sensitivity of the output. After a brief introduction to GEOFRAC the paper discusses the results of the simulations and attempts to explain how aperture, width and rotation (orientation distribution) of the fractures influence the resulting flow rate in the production well.

INTRODUCTION

In deep geothermal energy projects naturally and artificially induced fractures in rock are used to circulate a fluid (usually water) to extract heat; this heat is then either used directly or converted to electric energy. For this purpose the rock mechanics group in the Department of Civil and Environmental Engineering at MIT has developed the stochastic fracture pattern model GEOFRAC.

GEOFRAC is a three-dimensional, geology-based, geometric-mechanical, hierarchical, stochastic model of natural rock fracture systems (Ivanova, 1998). Fractures are represented as a network of interconnected polygons and are generated by the model through a sequence of stochastic processes (Ivanova et al., 2012). This is based on statistical input representing fracture patterns in the field in form of the fracture intensity P_{32} (fracture area per volume) and the best estimate fracture size E[A]. P_{32} can be obtained from spacing information in boreholes or from observations on outcrops using the approach by Dershowitz and Herda (1992). Best estimate fracture size can be obtained from fracture trace lengths on outcrops with suitable bias corrections as developed by Zhang et al. (2002). Distributions of fracture size can also be obtained subjectively. GEOFRAC has been applied and tested by estimating the fracture intensity and estimated fracture size from tunnel records and from borehole logs (Ivanova et al. 2004, Einstein and Locsin 2012). Since its original development, GEOFRAC has been made more effective by basing it on Matlab, and it has been expanded by including an intersection algorithm and, most recently, a flow model.

GEOFRAC belongs to the category of Discrete-Fracture Network models. In this type of model the porous matrix is not represented and all flow is restricted to the fractures. The flow rate is calculated by the Poiseuille relation for parallel plate flow:

$$\text{Qout} = \frac{w(s)\ h^3 \Delta P}{12\mu\ \Delta L} \qquad \textit{Equation 1}$$

Where:

- Qout (m^3/s) is the volumetric flow rate per unit width;
- h (m) is the aperture of the fracture;
- μ (kNs/m^2) is the fluid dynamic viscosity;
- ΔP (Pa) is the pressure change over the distance ΔL
- W (m): width of the fracture.

The conditions to apply the Poiseuille equation are:
- water flows only in the x direction between two parallel plates
- no-slip boundary conditions for viscous fluids forming the velocity profile in the y direction as shown in Figure 1.

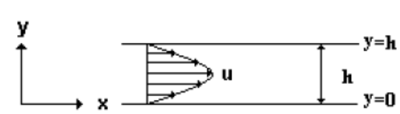


Figure 1 - Linear flow between parallel plates (no-slip boundary conditions)

As seen in Equation 1 width and particularly aperture affect flow. In the presented parametric analysis the effect of aperture is studied as well as fracture orientation distribution all of which contribute to the variation of the final results.

In this paper the basic concepts of the GEOFRAC model and the definition of all the parameters that are considered in this study are discussed first (Background). The paper then presents the parametric study, in which the effect of each parameter is investigated. This is done by considering the variation of one parameter at a time. The Conclusions summarize the outcomes of the parametric study.

BACKGROUND

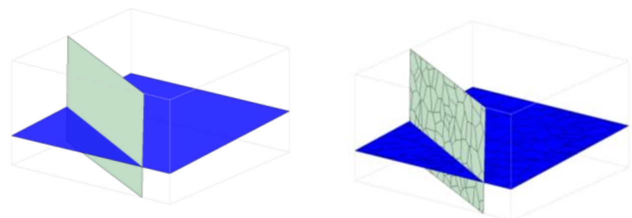
Fractures in GEOFRAC are represented as polygons. The desired mean fracture size E[A] and fracture intensity P_{32} in a region V are given as input. GEOFRAC uses these inputs to generate the fracture system following a sequence of stochastic processes (for details on GEOFRAC and on the Flow model see Ivanova et al. 2012, Sousa et al. 2012 and Sousa, 2013):

Primary Process: Fractures planes are generated in the volume V with a Poisson plane process of intensity μ where $\mu = P_{32}$. The orientation of the planes is specified with the Fisher distribution. Recall that this is a single parameter distribution. Low values of the parameter K simulate randomly generated planes; large K will generate planes mostly parallel to each other.

Secondary Process: A Poisson point process with intensity λ is generated on the planes and the fractures are created with a Delaunay-Voronoi tessellation. It represents fracture intensity variation by size and location.

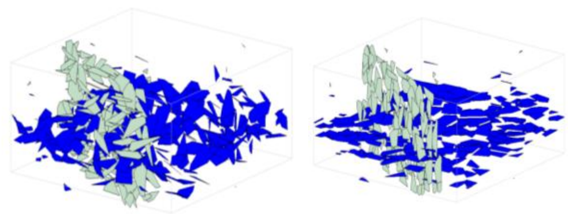
Tertiary Process: Random translation and rotation of the fractures (polygons) are conducted to represent the local variation of fracture position and orientation of individual fractures. In the tertiary process a new algorithm was recently added to the model to allow the user to model fractures with or without random rotation. The parametric study presented in this paper will compare results with rotation and no rotation of the fractures.

The generation process is visualized in Figure 2.



Primary Process: Generation of Planes planes into polygons

Secondary Process: Division of



Tertiary Process: Random translation and rotation

Figure 2 - Generation of a fracture set with the GEOFRAC model

Fractures intersect and create a path in which the fluid can flow. Each fracture has a particular width and length. When fractures intersect an average width w(s) of the intersection is calculated (Figure 3).

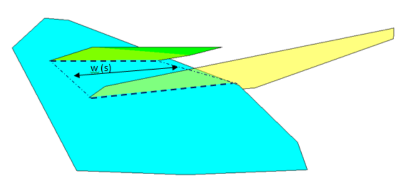


Figure 3 - Fracture intersections and the average width of the intersection

After the path from the injection well to the production well is calculated (Figure 4), fractures and paths are modeled as rectangular cross sections (**Figure 5**). Each path is composed of several fractures, and it can be represented by an equivalent fracture width (Weq), the aperture and the path length that is equal to the sum of the lengths of the fracture network. The Weq is computed as the weighted average of all the fractures that are part of the path.

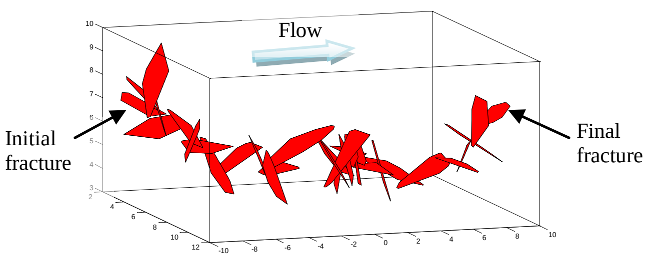


Figure 4 – 3D view of a simulated path in the controlled volume

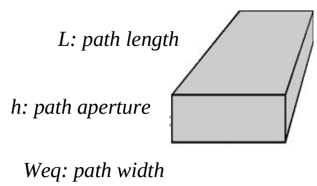


Figure 5 - Graphical representation of a path in GEOFRAC

In this study the effect of aperture, of the orientation of the planes represented by the Fisher parameter and of the orientation of the individual fracture (random rotation, no rotation) will be investigated.

PARAMETRIC STUDY

The parametric analysis was conducted in order to check the consistency of the model and determine which parameters have the greatest effects on the final results. The parametric analysis considers simplified conditions:

- A synthetic 10x10x10 m volume
- Injection and production wells are the boundaries of the volume
- The water temperature is assumed to be 20°C, i.e. the dynamic viscosity is 1.002×10^{-3} Pa s.

These simplifications are both justified and necessary since the parametric study intends to verify the intrinsic correctness of GEOFRAC flow model.
The results of this analysis are presented in the next paragraphs and are sub-divided into the following sections:

- output analysis varying the aperture parameter;
- output analysis varying the Fisher parameter that affects the orientation of the planes during the primary process;
- output analysis varying the rotation the fractures during the tertiary process.

Each section will conclude with a brief summary of the results obtained.

The following code will be used to identify the simulations done using the follow abbreviated indicators, in the charts as well as in some comments:

$$E[A] - P_{32} - K - R - h$$

Where:

$E[A]$ = Expected area of the fracture (m^2)
P_{32} = Fracture intensity (fracture area/volume)
K = Fisher parameter
R= 0 ‘no rotation’ of the fractures
R=1 ‘random rotation’ of the fractures
h = Aperture of the fracture (m)

Example: the code 2-3-1-0-0.005 means:
$E[A]=2\ m^2$, $P_{32}=3$, K=1, R=0 (no rotation), aperture=0.005 m

Output analysis varying the aperture parameter

GEOFRAC allows the user to generate the aperture (h) of the fractures using three models: a deterministic approach, in which the aperture is a function of the radius of the circumference that inscribes the fracture (polygon); a probabilistic approach, which follows a truncated lognormal distribution and a fixed value approach in which the users can fix the value of the aperture. This last approach was used for this analysis, in order to establish the sensitivity of the results and to confirm the direct relation between the aperture of the fractures and Qout (m^3/s). Two cases are analyzed: h = 0.005 m and h = 0.01 m. Most of the other parameters are kept fixed in this particular sensitivity analysis:
$E[A]= 2$
$P_{32} = 3$
K=1
R = 1 (rotation) or 0 (no rotation)

For both cases shown in Figure 6 (h = 0.005 m) and Figure 7 (h = 0.01 m), the results for no rotation and random rotation of the polygons are reported in the same chart. Just 40 simulations were run, a number considered representative since the aim of this analysis is to study the trend of the results and not to evaluate the exact value of Qout. A detailed study of sample analysis is presented in Vecchiarelli and Li (2013).

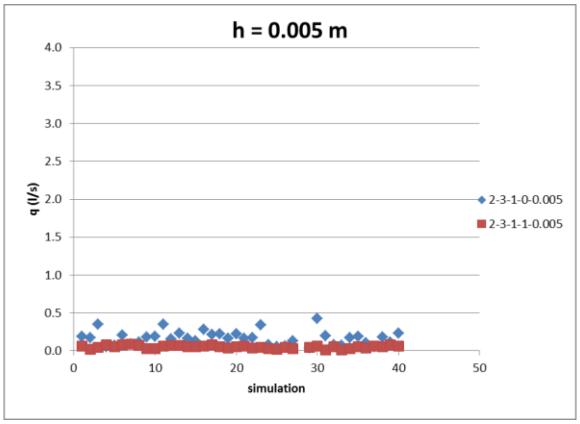


Figure 5 - Flow rate for h=0.005 m for no rotation and random rotation of the fractures

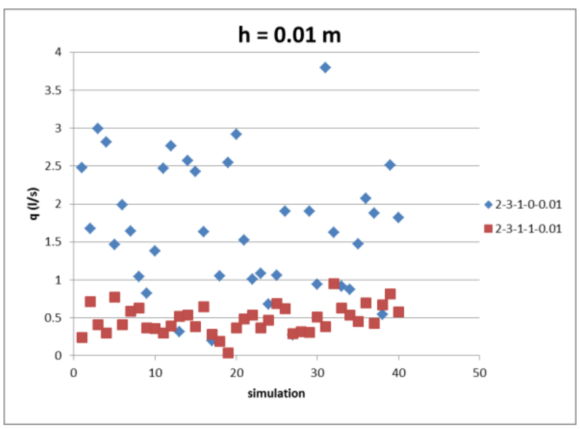


Figure 6 - Flow rate for h=0.01 m for no rotation and random rotation of the fractures

In Tables 1 and 2 the mean, the standard deviation and the coefficient of variation of the simulations are summarized.

Table 1: Values of Qout (m^3/s) for h = 0.005 m

Qout (m^3/s)	Mean	Standard Deviation	Coefficient of variation
no rotation	0.16	0.09	0.54
rotation	0.05	0.02	0.40

Table 2: Values of Qout (m^3/s) for h = 0.01 m

Qout (m^3/s)	Mean	Standard Deviation	Coefficient of variation
no rotation	1.64	0.85	0.52
rotation	0.48	0.19	0.38

The results correspond well to the theory. The ratio of the cubic value of the aperture parameters chosen for this analysis is 0.01^3/0.005^3=8. Qout for the rotation case for example is Qout= 5.16 m3/s for h=0.005 m and Qout=47.94 m3/s for h=0.01 m and producing a ratio of about 9 which is very close to 8. A similar ratio is obtained for the no rotation case. (The differences on absolute values for rotation and no rotation will be discussed later). The variability as expressed by the coefficient of variation is apparently not affected by the absolute value of h.

Output analysis varying the Fisher parameter

In order to check the variability of the results of Qout, using different Fisher parameters, two values were selected for the analysis presented in this section: K=1 and K=40. These two values represent the extreme conditions with K=1 representing randomly generated planes (**Figure 7**) and K=40 mostly parallel planes (**Figure 8**). The resulting Qout are plotted in **Figure 9 and 10.**

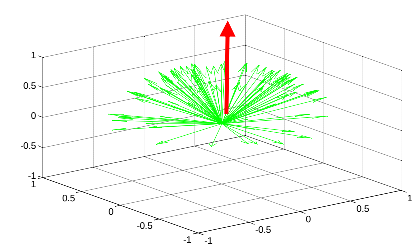


Figure 7 - Fracture set poles. Orientation distribution: Univariate Fisher K=1

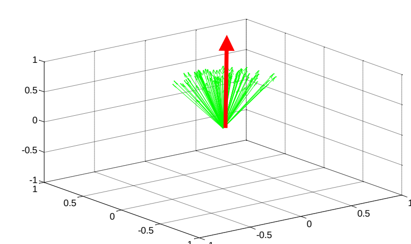


Figure 8 - Fracture set poles. Orientation distribution: Univariate Fisher K=40

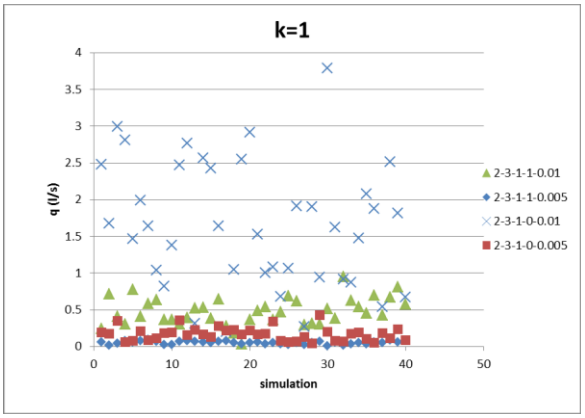


Figure 9 - Qout values for K=1

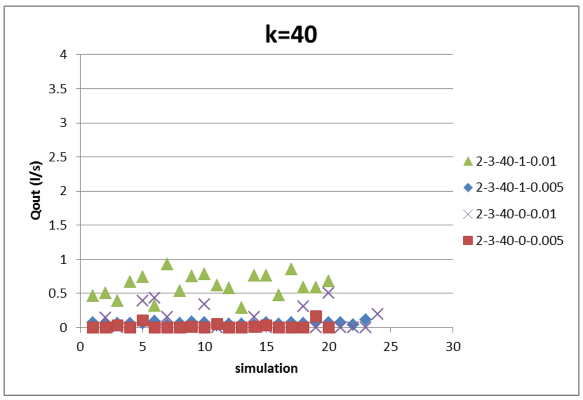


Figure 10 - Qout values for K=40

For K=1 (**Figure 9**) the largest values are obtained for h=0.01 m and no rotation. For K=40 this is different (**Figure 10**). In fact, for K=40 and h=0.01 the highest values of Qout occur for random rotation. A possible explanation is as follow: the rotation of the fractures starting from almost parallel planes (K=40) adds some randomness that increases the intersections between fractures generating more flow. On the other hand starting from random orientation of the planes (K=1) the fractures intersect because of the orientations of the planes with no rotation. The rotation appears to remove some of these intersections. In order to better understand this behavior the number of paths and their physical location in the control volume are shown in the following figures ***(Figures 11 to 18)*** in which the variation of all parameters is investigated (K=1, K=40; h=0.01 m, h=0.005 m; rotation, no rotation)

- *Plots of the paths K=1*

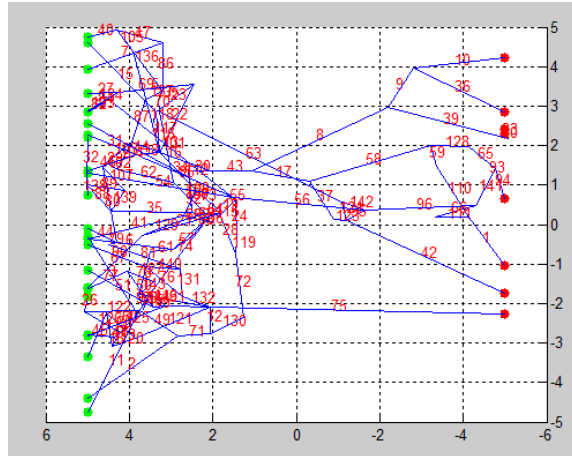


Figure 11 - h=0.005 rotation

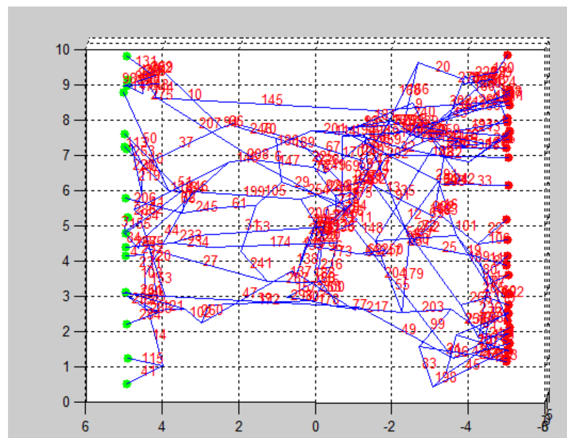


Figure 12 - h=0.01 rotation

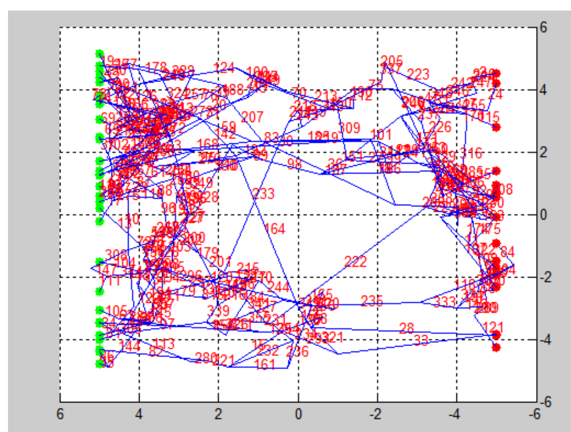


Figure 13 - h=0.005 no rotation

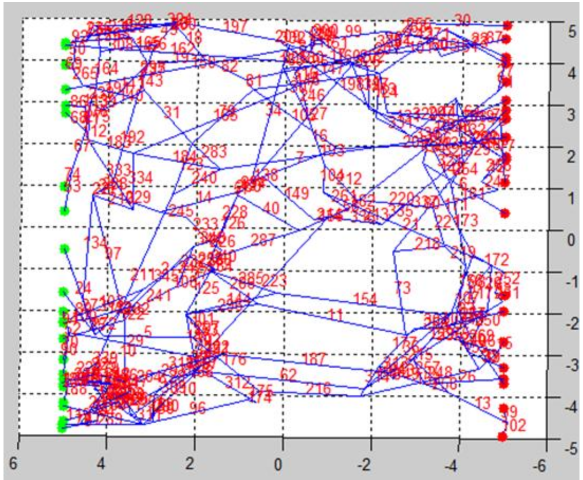


Figure 14 - h=0.01 no rotation

- *Plots of the paths K=40*

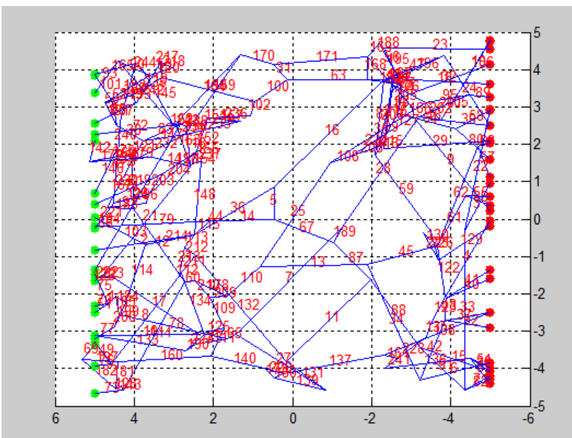


Figure 15- h=0.005 rotation

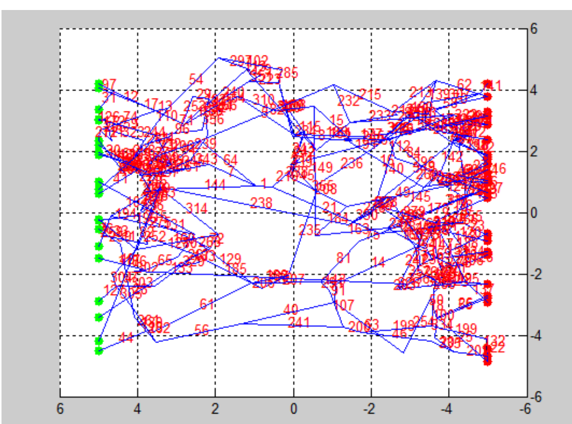


Figure 16 - h=0.01 rotation

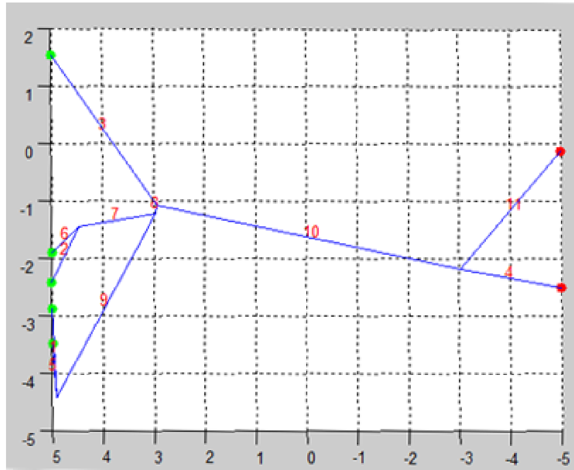


Figure 17- h=0.005 no rotation

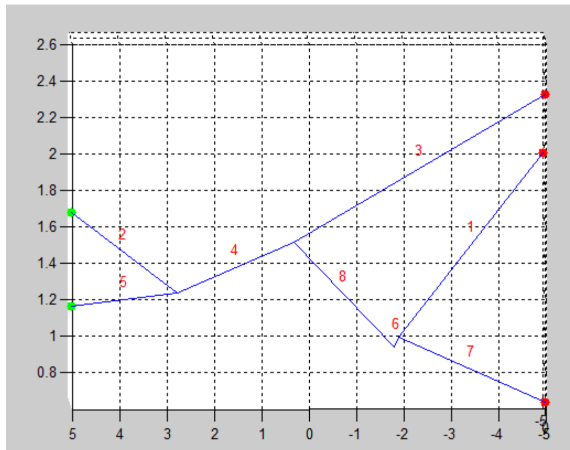


Figure 18 - h=0.01 no rotation

The following interpretation can be offered: for K = 1 (Figures 11 to 14) in general the number of paths is high. There are many fractures that intersect the two boundaries (green and red dots in the figures). Figure 11 (h=0.005, rotation) shows a somewhat odd behavior in that the number of paths decreases then increases again. This influences the flow, which in effect is smaller in this particular case. The h=0.01 m no rotation case (Figure 14) shows a higher number of paths and more branches compared to the other figures. This confirms the hypothesis made earlier that, with K=1 the planes in the volume are randomly oriented, and this produces a large number of intersections between fractures.

The results for K=40 paths are very consistent and clear. More branches occur in the rotation case (Figures 15 and 16) than in the no rotation case, (Figures 17 and 18) and as a consequence the number of paths is higher in the rotation case. As mentioned before the primary process for K=40 generates planes almost parallel to each other and the rotation of the fractures in the tertiary process increases the probability of intersections.

Effect of fracture translation

In the tertiary process fractures are translated and can be rotated or not rotated. The effect of rotation was discussed before. It is worthwhile to investigate what effect translation may have. Figure 19 shows the possible overlaps of apertures if the fractures are translated and not rotated. Such an overlap can produce a fracture path. However, when we investigated Qout as a function of translation and rotation as shown in Figure 20 one can see that translation has no effect both in the rotation- and the no rotation case and that the Qout for no rotation is higher (for reasons explained earlier). A possible explanation for the lacking effect of translation can be found with the numbers shown in Table 3. The minimum value of the translation is 0.016 m; so only few fractures with aperture h=0.01 m will overlap as shown in Figure 19.

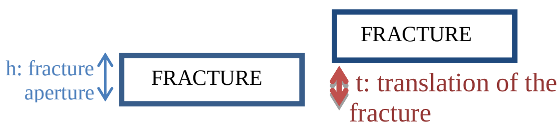


Figure 19 – Schematic representation of the translation between fractures

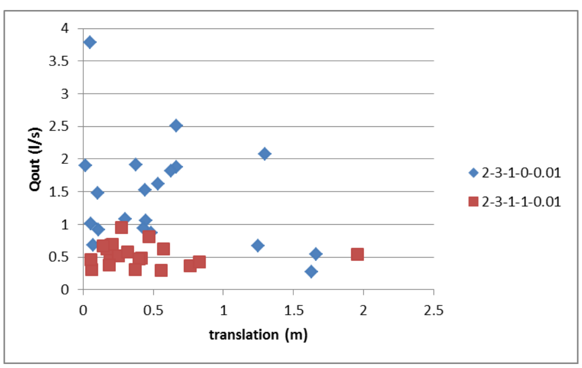


Figure 20 - Translation of the fractures vs Qout

Table 3 - Max and min values of the translation of the fractures

translation (m)	min	max
no rotation	0.016	1.659
rdm rotation	0.057	1.955

CONCLUSIONS

In this paper the results obtained from a simple parametric study with the fracture flow model GEOFRAC are analyzed and discussed. Recent developments have made GEOFRAC more efficient by basing it on Matlab, and it has been expanded by including an intersection algorithm and a flow model. The work in this paper represents an initial study with the flow model in which the influence of fracture aperture and orientation on the flow was investigated.

The parametric study demonstrates how aperture, the Fisher parameter and rotation of the fractures influence the production flow rate. As to be expected greater aperture produces greater flow. The effects of orientation are more complex as the effect of fracture plane orientation (Fisher parameter) and of rotation of individual fractures interact. For planes randomly generated the case with no rotation of the fractures and an aperture of 0.01 m generates greater flow than with rotation, while for parallel planes greater flow occurs in case of rotation of the fracture and aperture equal to 0.01 m.
This study of a simple synthetic case shows that the model is consistent but also that are some unexpected complexity affected by fracture orientation.

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REFERENCES

Dershowitz, W.S. (1984), “Rock Joint Systems,” *Ph.D. Thesis, Massachusetts Institute of Technology*.

Dershowitz, W.S.; Herda, H. (1992), “Interpretation of Fracture Spacing and Intensity,” *Proc. 33rd. U.S. Symp. on Rock Mechanics, Balkema*.

Einstein, H.H.; Baecher, G.B.; Veneziano, D. (1979), “Risk Analysis for Rock Slopes in Open Pit Mines,” *USBM Tech. Report*.

Einstein, H.H.; Locsin J.-L. Z., “Modeling rock fracture intersections and application to the Boston area” Journal of Geotechnical and Geoenvironmental Engineering, Vol. 138, No. 11, November 1, 2012

Ivanova, V. (1998), “Geologic and stochastic modeling of fracture system in rocks,” *PHD Thesis, MIT*.

Ivanova, V. and Einstein, H.H. (2004, "Three-Dimensional Hierarchical Stochastic Modeling of Rock Fracture Systems: an Example from the Yates Field," *In Proceedings: Gulf Rocks 2004, the 6th North America Rock Mechanics Symposium (NARMS), June 5 - 9, 2004, Houston, Texas*.

Ivanova, V., Sousa R., Murrihy B., and Einstein, H.H. (2012), "Mathematical Algorithm Development and Parametric Studies with the GEOFRAC Three-Dimensional Stochastic Model of Nature Rock Fracture Systems" *to be submitted.*

Sousa, R.L.; Ivanova, V.; Murrihy, B.; Einstein, H.H. (2012). Decision Aids for Engineered Geothermal Systems: Fracture Pattern Modeling. Research report, Massachusetts Institute of Technology, Cambridge, MA.

Sousa, R. (2013), "Three-Dimensional discrete fracture flow model". Research report, Massachusetts Institute of Technology, Cambridge, MA (draft).

Vecchiarelli A., Li W. (2013) "Parametric study with GEOFRAC", *MIT,* research report *draft*