NON-LINEAR MODELLING OF A GEOTHERMAL STEAM PIPE NETWORK

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SUMMARY - Recent work on developing a non-linear model for a geothermal pipe network system is presented in this paper. Previous methods of modelling are reviewed. Using the mass and energy balance at each node and loop of a pipe network, a non-linear equation set can be established. The equation set is formulated from a conceptual model of the pipe network, which is composed of steam wells and connected pipelines. The well characteristics curves, pipeline frictional characteristic and the different combinations of the connected components all contribute to the pressure and mass flow rate distributions in the system. Some recent numerical experiments on the non-linear model are compared with Italian experience. The results converged satisfactorily. Using this model, the user can change the connection and the characteristic of each individual demand in a network and chose a desired manifold pressure; the simulator will then calculate the balanced mass flow rate and pressure distribution along the defined pipe network.

1. INTRODUCTION

Computer modelling of geothermal pipe networks for both design and simulation purposes has been reported. Most of these are linear computing models, which can work well on a tree-like linked simple pipe network. However with more complex practical systems, it is easy to find that both loops of pipelines and a series of non-linear well characteristics have to be considered. These two factors contribute to the difficulty of the modelling work. Loops in a pipe network can normally be solved using a number of established methods (Stephenson, 1989). However if the pipe network is connected to a number of wells with parabolic like characteristic curves rather than linear curves, even the calculation of a simple network can be a cumbersome process. Convergence of the solution is often very slow because of the trial and error methods used.

This paper presents recent development work on a numerical model which uses a non-linear method to simulate a geothermal steam pipeline network. The mass and pressure balance at each node and loop of a pipe network are considered to be under the control of both the connected well and the manifold working point. After setting up a non-linear equation set for the above balance, numerical methods are used to solve for the mass flow and pressure distribution. Convergence of the solution has been satisfactory for the numerical experiments conducted. In this paper, a recent simulation on one of the longest networks at Larderello, Italy, is presented. The simulation results are compared with field measurements and an earlier simulation by Marconcini and Neri (1979). Sensitivity of the simulation to the pipeline loss factor and well characteristics are discussed.

Since the non-linear model solves the set of equations describing a network simultaneously, it has good flexibility, making it particularly applicable to looped networks which are much more difficult to solve than a tree-like network.

2. NON-LINEAR MODELLING STRATEGY

A non-linear model solves the equation set simultaneously using a numerical method formulated from the conceptual model of the pipe network. A computer code has been developed based on this non-linear modelling strategy. The simulation results for a series of test models have shown a satisfactory convergence of the solution.

2.1 Review

The pressure and mass flow rate distribution through a pipe network are generally controlled by the pressure difference between the input and output points of the system. In a steady-state operation, the mass flow and pressure drop at each node and loop should be balanced that is:

- the net flow towards any junction or nodes is ZERO, and
- the net head loss around any closed loop is ZERO.

Head loss along a pipe line are usually assumed to be of the form

\[ Ah = \frac{kLQ^p}{D^q} \]

where \( Ah \) is the head loss, \( k \) is the friction factor, \( L \) is the pipe length, \( Q \) the volume flow rate and \( D \) the internal diameter of the pipe. Most methods of network analysis are based on the above equation (Stephenson, 1989).

Two early approaches to pipe network analysis were the Loop Flow Correction Method and the Node Head Correction Method. Both methods use successive correctors speeded by a mathematical technique developed by Hardy Cross (1936). The development of micro computers has made it much easier to perform a network analysis by numerical methods. This involves the simultaneous solution of equations describing flow and pressure balance. When the inputs to a network system are steam wells with parabolic characteristics, a non-linear numerical solution is a requirement of the model.
2.2 Non-linear Modelling

A conceptual model which can represent the real pipe network is the first requirement for setting up a non-linear equation set. An effective numerical method is necessary for an accurate solution with quick convergence.

2.2.1 Conceptual Model

Before pressure and mass balances are applied at each loop and node, a conceptual model is required. This model should reflect all the interrelations between each part of the network and should be concise and easy to use. Fig. 1 shows a typical conceptual model of a network where \( P_1(m_1), P_2(m_2) \) and \( P_3(m_3) \) indicate the characteristic curves of the three steam wells, \( P_T \) indicates the required steam pressure at the manifold, \( m_1, m_2, m_3, m_4 \) and \( m_5 \) are the stable-state mass flows in the network, and the nodes are numbered as 1, 2, 3, 4, 5 and 6. Dummy lines, represented by dashed line are used between each input and output point of the system. With the help of these dummy lines, the network is linked by a number of enclosed loops on which the pressure balance rule of a loop can be applied. The expected flow directions are marked on each pipeline. The following conventions are then applied to establish an equation set for the network.

- At a balanced node the input flow is positive, output flow is negative.
- In a balanced loop, an arbitrary calculation direction of the loop is assumed. If a well output has a same direction as the calculation direction the well head pressure \( P_i(m_i) \) is positive, otherwise it is negative; the opposite rule is applied for output pressure \( P_T \) in the steam manifold.
- If a pipe flow \( m_i \) has a same direction as the calculation direction, the frictional pressure drop is positive. Otherwise it is negative.
- In the dummy pipe, the mass flow rate is defined as zero.

\[
\begin{align*}
\text{m}_4 \cdot \text{m}_2 \cdot \text{m}_3 & = 0 \\
\text{m}_5 \cdot \text{m}_1 \cdot \text{m}_4 & = 0 \\
P_1(\text{m}_1) - P_1(\text{m}_1) - P_5(\text{m}_5) - P_T & = 0 \\
P_2(\text{m}_2) - P_2(\text{m}_2) - (m_4 + m_5) - P_T & = 0 \\
P_3(\text{m}_3) - P_3(\text{m}_3) - (m_4 + m_5) - P_T & = 0
\end{align*}
\]

From equation (1) and (2) it is obvious that the equation set (3) is a non-linear set and should have a simultaneous solution for \( m_1, m_2, m_3, m_4 \) and \( m_5 \), if they exist.

2.2.2 Establishing the equation set

In the conceptual model of Fig. 1, well head pressure can be expressed as:

\[
P_i(m_i) = A_i + B_i m_i + C_i m_i^2
\]

where A, B and C are regression coefficients of the well characteristic curve and \( m \) is mass flow of the well.

Frictional pressure loss along the pipelines is given as:

\[
F_i(m_i) = K_i m_i^2
\]

where \( K \) is the loss factor on a section of a pipeline.

A system constraint is the requirement of a fixed output pressure \( P_T \) at node 6, so the mass and pressure balance equation set can be written as:

\[
\begin{align*}
\text{m}_4 \cdot \text{m}_2 \cdot \text{m}_3 & = 0 \\
\text{m}_5 \cdot \text{m}_1 \cdot \text{m}_4 & = 0 \\
P_1(\text{m}_1) - P_1(\text{m}_1) - P_5(\text{m}_5) - P_T & = 0 \\
P_2(\text{m}_2) - P_2(\text{m}_2) - F_4(\text{m}_4) - F_5(\text{m}_5) - P_T & = 0 \\
P_3(\text{m}_3) - P_3(\text{m}_3) - F_4(\text{m}_4) - F_5(\text{m}_5) - P_T & = 0
\end{align*}
\]

For a nonlinear equation set such as

\[
f_i(x_1, x_2, \ldots, x_n) = 0 \quad i = 1, 2, \ldots, n
\]

We can define an objective function

\[
F(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} f_i^2(x_1, x_2, \ldots, x_n)
\]

When we have

\[
F(x_1^*, x_2^*, \ldots, x_n^*) < \varepsilon
\]

then \( x_1^*, x_2^*, \ldots, x_n^* \) are roots of the nonlinear equation set (4).

Programming for the above process is performed as follows:

1. Starting from an initial guess of non-zero roots \( x_1^0, x_2^0, \ldots, x_n^0 \), suppose the iteration has progressed to the Kth step, then we have \( x_1^k, x_2^k, \ldots, x_n^k \);

2. Calculate the objective function \( F(x_1^k, x_2^k, \ldots, x_n^k) \).
3. If $F(x_1, x_2, \ldots, x_n) \in E$, then $x_1, x_2, \ldots, x_n$ is taken as a solution, otherwise go to next step;

4. Calculate:

$$\frac{\partial F}{\partial x_1} = \frac{F(x_1 + \Delta x_1, x_2, \ldots, x_n) - F(x_1, x_2, \ldots, x_n)}{\Delta x_1}$$

$$\frac{\partial F}{\partial x_2} = \frac{F(x_1, x_2 + \Delta x_2, \ldots, x_n) - F(x_1, x_2, \ldots, x_n)}{\Delta x_2}$$

$$\frac{\partial F}{\partial x_n} = \frac{F(x_1, x_2, \ldots, x_n + \Delta x_n) - F(x_1, x_2, \ldots, x_n)}{\Delta x_n}$$

where $\Delta x_i = C x_i$ if $x_i \neq 0$

or $\Delta x_i = C x_i$ if $x_i = 0$

5. Calculate:

$$x_i^{k+1} = x_i^k - \lambda_k \frac{\partial F}{\partial x_i}, \quad i = 1, 2, \ldots, n$$

where $\lambda_k = \sum_{j=1}^{n} \frac{\partial F}{(\partial x_j)^2}$

then repeat from step 2 until convergence is obtained at step 3.

A computer code, written in TURBO PASCAL, for the solution process has been developed (Huang, 1991).

2.3 Test Model

In order to test the computer program, well data from the geothermal field has been used in a test model. The operation of the program and the interrelations between parameters (Huang, 1992) have been investigated. Different connections, using the same basic model were simulated in order to compare results for different operation conditions.

2.3.1 Test Pipe Network

The basic model is shown in Fig. 2. The model is composed of four production wells, a single-looped branch line network between the wells, and a main line connecting the network to a power house (steam consumer). With this model, it is shown how a nonlinear looped network is solved, although such connections may not be necessary in practice.

In the layout of the conceptual model of this network, $m_1, m_2, m_3,$ and $m_4$ represent the steam mass flow from wells 1, well 2, well 3 and well 4 respectively. All these mass flows are controlled by the well and separator characteristics $P_1(m_1), P_2(m_2), P_3(m_3)$ and $P_4(m_4)$.

However, the frictional pressure loss along each line also make a contribution to the control of all the mass flow rates from $m_1$ to $m_9$. This frictional pressure loss is described by $F_i = K_i(m_i)^2$. The friction loss factor is defined as:

$$K_i = \frac{32 f L_i}{10^4 \pi^2 D^5 \rho}$$

where $f$ is Fanning friction factor (Perry, R. H., 1987), $L_i$ and $D$ are the pipe length and inside diameter, and $\rho$ is the density of the steam.

The turbine inlet steam pressure is set at $P_T = 13.5$ bara. The pipe network has nine unknown variables, i.e. $m_1$ to $m_9$, and nine nonlinear equations can be established. A constant $K_i$ value is used for this test model.

2.3.2 Test Model Results

Using the model described above, a series of test runs were made with different combinations of the working wells used to simulate the possible different working conditions. Each combination followed the law that if one well is shut off, the corresponding linking branch lines are also shut off.

All the simulations converged satisfactorily. Results are shown in Table 1. The network output steam is used as the input for Turbine and the system output is given in terms of Turbine output in MW. This numerical experiment on the non-linear model illustrates that a pipe network can be simulated for any combination of wells on-line and different pipe frictional characteristics.
The results of these simulations can be used to predict the general performance of a non-linear pipe network system when

(a) the number and order of the wells on-line have been changed (some wells shut off);
(b) the frictional characteristics of any pipeline have been changed (valving);
(c) the characteristic curve of any production well changes due to long term operation.

3. SIMULATION OF A STEAM PIPE NETWORK

The non-linear model was applied on one of the longest steam pipe networks operating at Larderello, Italy and the results compared to published data. The conceptual model of the network and the simulation results are presented.

3.1 Network Layout and Conceptual Model

This network carries fluid from Puntone 1, Querciola 2, Capriola, Grottitana and VC 2 wells to the Serrazzano power plant. Fig. 3 illustrates the layout of the pipe system. Several condensate dischargers are placed along the line. The characteristic curves of the different wells were calculated from published data. The loss factor, \( K \), is based on the geometry of each pipeline and the corresponding steam state. A conceptual model for the network is shown in Fig. 4.

### Table 1 System performance with different wells on-line

<table>
<thead>
<tr>
<th>Model</th>
<th>Well 1 (BR 2)</th>
<th>Well 2 (BR 3)</th>
<th>Well 3 (BR 11)</th>
<th>Well 4 (BR 17)</th>
<th>Turbine (1.5/3.5 bar.n)</th>
<th>Input (Kg/s)</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>M (Kg/s)</td>
<td>14.023</td>
<td>12.355</td>
<td>10.332</td>
<td>8.015</td>
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</tr>
<tr>
<td>RMFC</td>
<td>1.824</td>
<td>1.541</td>
<td>1.314</td>
<td>1.060</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M (Kg/s)</td>
<td>12.650</td>
<td>10.791</td>
<td>8.267</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMFC</td>
<td>1.540</td>
<td>1.106</td>
<td>1.066</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 2-3</td>
<td>12.652</td>
<td>12.628</td>
<td>19.276</td>
<td>2.96</td>
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<td>M (Kg/s)</td>
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<td>8.419</td>
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<td>1.066</td>
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<tr>
<td>Model 2-4</td>
<td>13.552</td>
<td>8.481</td>
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<td></td>
</tr>
<tr>
<td>M (Kg/s)</td>
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<td>8.235</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMFC</td>
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<td>1.006</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Model 2-5</td>
<td>13.963</td>
<td>13.935</td>
<td>34.010</td>
<td>5.20</td>
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</tr>
<tr>
<td>M (Kg/s)</td>
<td>15.003</td>
<td>10.774</td>
<td>8.235</td>
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<tr>
<td>RMFC</td>
<td>1.822</td>
<td>1.109</td>
<td>1.006</td>
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<tr>
<td>Model 2-6</td>
<td>14.053</td>
<td>14.031</td>
<td>14.018</td>
<td>25.082</td>
<td>5.46</td>
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<tr>
<td>M (Kg/s)</td>
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<tr>
<td>RMFC</td>
<td>1.800</td>
<td>1.109</td>
<td>1.006</td>
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<tr>
<td>Model 2-7</td>
<td>13.747</td>
<td>13.734</td>
<td>23.597</td>
<td>3.42</td>
<td></td>
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</tr>
<tr>
<td>M (Kg/s)</td>
<td>12.525</td>
<td>8.268</td>
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<td></td>
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</tr>
<tr>
<td>RMFC</td>
<td>1.304</td>
<td>1.066</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 2-8</td>
<td>13.946</td>
<td>13.936</td>
<td>31.702</td>
<td>4.85</td>
<td></td>
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</tr>
<tr>
<td>M (Kg/s)</td>
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<td>10.791</td>
<td>8.267</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMFC*</td>
<td>1.822</td>
<td>1.541</td>
<td>1.314</td>
<td>1.060</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Maximum Relative Error of RMFC: 0.22% 6.52% 0.76%

Note: RMFC - Relative Mass Flow Contribution
3.2 Results

A non-linear numerical simulation of the Larderello pipeline network was successfully performed with three different sets of input data in TEST 4, TEST 5 and TEST 6. The iterative procedure takes about 5 CPU minutes to converge on a IBM PC 386 computer. The results were printed as the mass flow rate from $m_1$ to $m_2$. The required manifold pressure was taken from published data (Marconcini and Neri, 1979). In TEST 5, the K value for pipelines between well Capriola and VC 2 has been modified by an increase shown as follows:

<table>
<thead>
<tr>
<th>Test</th>
<th>$K_{4.5} + K_{5.6} + K_e$</th>
<th>$K_7-8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEST 4</td>
<td>0.00024</td>
<td>0.00020</td>
</tr>
<tr>
<td>TEST 5</td>
<td>0.00037</td>
<td>0.00030</td>
</tr>
</tbody>
</table>

In TEST 6, a constant Fanning friction factor $f=0.0033$, which is an average value for geothermal piping (Huang, 1992), was used in the calculation of all the K values throughout the network. The mass flow rate for each well from the results is used in the well characteristic curves to give the operating wellhead pressures. Pressure profiles are then plotted over the measured data along the pipeline network, as shown in Fig. 5. The resultant curves representing TEST 4, TEST 5 and TEST 6 fall within a close range of the measured data irrespective of the difference in input data. Considering the simplification made, the result is good enough to show an agreement between the numerical simulation and the measurements.

4. DISCUSSION

One of the sensitive parameters in the simulation is the loss factor K. Improperly used K values in the system may lead to a non-convergent simulation. Fortunately, the K value found in practice is dependent on the change of Fanning friction factor $f$, which happens to fall in a narrow range. Different tests have shown that the simulation results for pressure drop are not too sensitive to Fanning friction factor $f$.

In Fig.5, the comparison between the simulation and measurement is illustrated. The narrow gap between the two measurements may indicate measurement errors. Most of the simulation results fall within the range of measurement error, which indicate a good agreement with measurement. All three simulations, TEST 4, TEST 5 and TEST 6 have a slightly flatter pressure profile along the pipeline than the measured data. This is because localized frictional loss has been neglected on the simulation at this stage. For the results for TEST 4, the pressure profile has a similar slope to the measurements except for the pipelines from node 4 to 8. This might be due to some addition localized frictional loss. In TEST 5, the K value for all pipelines has been modified by a small increase. The simulation result shows that the slope of the pressure profile between node 4 and 8 is closer to the measured one, demonstrating the sensitivity to the K value.

TEST 6 is based on a constant Fanning friction factor $f$ of 0.0033. The objective of this test was to investigate the sensitivity to $f$. It is interesting to note that the resultant pressure profile of this test is very close to that of TEST 4. This is an indication that the $f$ value, if within a reasonable range, is not a sensitive parameter in this numerical simulation. Among the variables involved in evaluating K, pipe diameter is the most sensitive one for estimating pressure drop. Fortunately it is one of the most well specified parameters.

Of the main parameters, production well characteristic curves are of special importance. These control the general performance of a connected pipeline network, though the change of the frictional characteristic of the network can bring a change in the general performance to some extent.

It is the well curve which controls how much mass flow the well contributes to the network. Since each well may have its own characteristic curve, any pressure change along the pipeline can cause a corresponding change of the working point of the well along its production curve. As a result, the well production rate is changed which then leads to a consequent change in pressure drop on the network. Needless to say, this can also bring an influence on the working points of other wells and their production rates.
The influence on the other well operating points of changing a well characteristic curve is very complex. It is a function of their curve shape, the frictional characteristic of each pipeline and the mass and energy balance of the whole network. In some cases, it can cause an increase in the total system output while in some other cases, an opposite effect occurs. Generally speaking, a cluster of wells having similar characteristic curves tend to give a simpler network which is easy to control. With the help of the simulation, well head control of the working point can be used as a systematic control to a pipe network, especially after there has been a change to the network system. To examine more detailed interrelations between wells, a study of different combinations of wells and the corresponding simulation results are needed.

At this stage, the calculation in the mathematical model is based on saturated steam. Heat loss along the pipeline has not yet been taken into account. The results from simulation TEST 6 have shown little difference from the published results of another simulation VAPSTAT 1 (Marconcini and Neri, 1979) as illustrated in Fig. 6.

The similarity of the two simulation results indicates that the superheat of the steam and the heat loss along the pipeline play little part in the numerical simulation. In VAPSTAT 1 about 80°C superheat of the steam is considered. Heat losses through the pipeline surface are also included. From steam tables, it can be seen that 80°C super-heating causes an 18% change in density of steam at a pressure of 8 bars. This small change seems to cause very little change to the \( f \) value and the consequent pressure drop. Since the pipeline is insulated, the temperature drop along the pipeline is small. This small reduction of the superheat has little effect on the pressure drop.

5. CONCLUSIONS

1. A non-linear model for the numerical simulation of a geothermal steam pipe network has been developed. A mass flow and pressure balance at each node and loop in the network is used to establish a non-linear equation set. Well characteristic curves normally dominate the distribution of the mass flow within a pipe network and the total output.

2. With the successful numerical simulation of one of the longest steam pipe networks in Larderello, the non-linear model has been validated. Superheating of the steam and the heat loss through pipeline surface are not calculated at this stage. The simulation converged with results close to the measured data within 5 minutes CPU time. The flatter slope of the simulated pressure profile along the pipeline is caused by neglecting the localized frictional loss.

3. Different simulation results have shown that they are not sensitive to the Fanning friction factor \( f \). This may indicate the possibility of using a constant or linear (against pipe roughness \( \varepsilon/D \)) \( f \) value for a simplified calculation of pressure drop. The correlation used for frictional pressure drop is proportional to the pipe diameter to the minus fifth power, the diameter \( D \) becomes the most sensitive factor in the calculation.

4. With the non-linear model, the simulation of a pipeline network can be applied not only to a tree-like system, as for VAPSTAT 1, but also to a looped one. The non-linear model allows changes of conditions in the network system and changes to the manifold pressure to be made at the discretion of the user.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

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