Finite Element Method (FEM) Geocomputing is increasingly important in analysing and evaluating enhanced geothermal reservoirs. Due to the complexity of geological objects, it is difficult to build a reasonable mesh for FEM. The development of digital 3D geological modelling tools and applications (such as GeoModeller and GoCad) makes it possible to establish geometric models for 3D geological objects. Our work focuses on constructing a geothermal reservoir system which transforms geometric models used in 3D geological visualization into FEM models. A 3D Geological model for visualization is used as input here, and the transformation processes are implemented following: (1) generating a tetrahedral mesh by 3D Delaunay triangulation methods; (2) abstracting boundaries of different geological objects; (3) building a well model by tetrahedral mesh refinement, which will be used in future FEM-based simulations.

Keywords: GeoModels; Delaunay Triangulation; 3D geological modelling; FEM

1. Introduction

In the field of geoscience, modelling of geological objects is of great interest in visualization and simulation. With the development of digital 3D geological modelling applications (such as GeoModeller and GoCad), 3D geological objects including geological units, faults and ore deposits can be easily built into GeoModels. However these GeoModels are mainly used for visualization, and lack topological relations which are crucial in simulation. Topological query [1-3] is a common issue which depends on topological relations between different geological objects. Furthermore, in the field of analysing, topological models which are suitable for simulating are urgently requested. The Finite Element Method (FEM), which is based on the FEM mesh, is an effective means of geocomputing. Whenever 3D geological objects are transformed into an FEM mesh such as a tetrahedral or hexahedral grid, FEM can be employed to present simulations for future analysing and evaluating. 3D Delaunay triangulation (DT) [4, 5] is one well studied 3D tetrahedral mesh generation method, which can be utilized to transform geometric geological models into FEM ones. Recently, regarding to DT, scientists [1, 6, 7] have worked on modelling geological objects innovatively. Ledoux [6] used the dual of DT named Voronoi Diagram (VD) to represent and analyze oceanographic datasets. Ledoux [7] also compared the VD-based method with geographical information systems (GIS) and outlined shortcomings of GIS and merits of the VD approach. Pouliot [1] used DT to build topological relationships between geological objects.

Our work focuses on constructing a geothermal reservoir system by transforming geometric geological models into a FEM mesh which will be utilized in simulation by FEM. Generally speaking, the output of geological modelling tools can be formed into a set of triangles which respect the geometric profiles of geological objects. Taking these triangular surfaces as input, our transformation process is outlined as below: In section 2, automatic DT mesh generation based on 3D geological modelling data, including boundary abstraction of different geological objects. In section 3, one of the geological objects is chosen to illustrate how to build a well model based on tetrahedral mesh refinement, and a boundary condition will be introduced for FEM in future analysing. Finally, conclusion and further work are discussed in section 4.

2. Topological modelling for 3D geological objects

2.1 3D geological objects data input and refinement

In this paper, we choose outputs of software such as GeoModeller and GoCad as input data. It is a set of geological objects, including two faults through different geological units, as illustrated in figure 1. The model data are in the format of a set of triangular facets which respect boundaries of different geological objects. The beginning of this file, node coordinates are provided. And then boundary triangular facets are presented by the indexes of nodes.
As our approach to extracting boundaries between different geological objects is based on density of boundary nodes, the geological model should have a set of reasonable discretized nodes on its boundaries. In other words, nodes used to represent boundaries of the geological model should have a specified distance $\epsilon$ between each other. This distance $\epsilon$ is defined as the precision of our transforming process.

The Edge-splitting method, which breaks a triangular edge into two subordinate ones, is employed to refine the initial geological model boundaries. It is simple but effective, as illustrated in figure 2. The middle point P of Edge AB is introduced to split AB into two edges AP and PB. And then triangles linked with AB are separated into two triangles respectively.

2.2 3D Delaunay triangulation

After refining the boundaries of the geological model, nodes on boundaries appear with a reasonable density due to a specified $\epsilon$. In order to distinguish different parts of the geological model and construct a topological model for FEM, a tetrahedral generation method should be presented on the current boundary nodes of the geological model. Because nodes that are close to each other (which respect model boundaries) are expected to construct tetrahedrons, then the Delaunay triangulation method is adopted to generate the tetrahedral mesh for the geological model.

First of all, the definition of Delaunay triangulation is introduced below:
Let $\{P_k\}$ be a set of points in $\mathbb{R}^d$. The Voronoi cell of $P_k$ is $V_k$ defined by

$$V_k = \{ p : \|p - P_k\| \leq \|p - P_j\| \quad j \neq k \}.$$ 

forms the well-known Voronoi Tessellation of the entire domain concerning $\{P_k\}$. The Delaunay triangulation of $\{P_k\}$ is defined as the dual of the Voronoi Tessellation; it was presented in 1930 by Delaunay, who found the property of empty circumsphere criterion. In the 3D domain, the empty circumsphere criterion can be presented as below: for every tetrahedron there are no vertices in its circumsphere except for the four vertices of its own. Then in 1981, Bowyer[4] and Watson [5] proposed a method of this algorithm, which is simple and efficient. The kernel of this method is inserting points step by step. In order to keep the empty circumsphere criterion, correct the topology of the mesh after inserting a point. Algorithm 1 shows the basic procedures of this method used in this paper.

**Algorithm 1: Classical 3D Delaunay triangulation**

**Step 1:** Construct a super tetrahedron, which contains all points prepared to be inserted. This super tetrahedron bounds a convex domain.

**Step 2:** Insert a point into this convex domain, and extract all the tetrahedrons violating the empty circumsphere criterion.

**Step 3:** Delete tetrahedrons extracted in Step 2, which leads to construction of a convex cavity. Use the inserted point to generate tetrahedrons with the surface of this cavity.

**Step 4:** Repeat Steps 2-3 until all the points are inserted.

2.3 Abstraction boundaries from different geological objects

As 3D DT tends to generate tetrahedrons by nodes close to each other, most of the triangular geological surfaces will construct naturally during the process of DT. Mark facets which do not have any edges longer than $\varepsilon$ (which is specified in section 2.1) boundaries. And then use Algorithm 2 to abstract boundaries of different geological objects.

**Algorithm 2: Abstraction of geological object boundaries**

**Step 1:** Roughly abstract geological objects in the form of tetrahedrons by facets which are marked as boundaries.

**Step 2:** Allocate each geological object a universal ‘colour’, and calculate its volume by its components (tetrahedral elements).

**Step 3:** Sort geological objects by their volumes in an increasing order.

**Step 4:** Extend geological objects to their neighbours one by one, merging the gap generated by boundary facets.

**Step 5:** Abstract facets shared by tetrahedrons with different colours.

In our geological sample, due to two faults, after abstraction the whole model (as illustrated in figure 4 (a)) is separated into 25 components (in figure 4 (b)), and colours of geological objects are updated, so they appear with colours different from original ones. Boundaries between different components have abstracted in the form of triangular facets which contain topological relationship information. Because the number of boundary triangular elements is too huge to visualize clearly, only one of the 25 components which is a corner of the geological model is illustrated in the format of triangular boundary, as shown in figure 4 (c). Furthermore, two faults break the geological model into three blocks, as shown in figure 4 (d) (e) and (f), which is useful for further simulation and analysing. With the help of the topological model generated above, different geological objects can be easily identified and the generated tetrahedral mesh is an ideal model for FEM-based simulation.
3. Refining geological object and building well model

In the field of geothermal exploration, wells are essential facilities. Simulation based on FEM methods requests elements with small size around wells, which can enhance the accuracy of analysing. To meet this need, first of all, a component which is suitable for drilling is chosen by experienced engineers (take a component which is far from faults for example, as illustrated in figure 5 (a)), and then the 3D geological model is refined in the following steps: (1) choose a particular position which is prepared to create the well model, as shown in figure 5 (b); (2) refine the geological object normally, as shown in figure 5 (c); (3) according to a specified precision, restrict the element size around the well; (4) refine the well in the geological model based on element size restriction, as shown in figure 5 (d). Finally, a desired tetrahedral mesh which is used in future FEM simulation and analysing is obtained, and the local tetrahedral mesh is illustrated in figure 5 (e).
4. Conclusion and future work

In this paper, automatic meshing and construction of a geothermal reservoir system is proposed, which focuses on transforming geometric models into an FEM mesh. In order to make a general interface between geological softwares (such as Geomodeller and GoCad) and the geothermal reservoir system, the input data are designed in the form of a set of triangles which presents the geometric features of the geological model. Then in the process of transforming, input data refinement, tetrahedral mesh generation and geological objects boundaries abstraction are performed step by step, which focuses on generating a reasonable FEM mesh. Finally, the well model used in geothermal exploration is introduced; automatic mesh generation for it is demonstrated in detail. Our approach presents a way of using geological data for visualization to construct an FEM mesh for geothermal simulation.

Further works should be done to enhance the approach proposed in this paper: in the first place, instead of a specified precision $\varepsilon$, boundary abstraction should be improved to adaptively detect small features on geological models; in the second place, as geological models have been divided into several parts, both parallel mesh generation and FEM analysis should be considered.

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