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A Connectivity-Graph Approach to Optimising Well Locations in Geothermal Reservoirs

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ABSRACT

In this work we address the problem of optimal location of injection and production wells in fractured-based geothermal reservoirs. The optimisation is based on a distance distribution function, and the length and aperture of pathways between the two wells. The initial locations of the two candidate wells are chosen at random and the fracture pathways between the wells are determined using graph theory concepts. The weighted shortest pathway incorporates the equivalent aperture and total length of pathway elements (i.e., linked fractures). The method is efficient and effective for generating final optimal well locations (as coordinates) and also provides a map of optimality for any given fracture network. The sampling scheme used can incorporate any constraint including technical, topographical and or design. Furthermore, stochastic modelling of fracture networks can be used to extend the use of the proposed method to deal with the uncertainty involved in estimated or simulated fracture networks.

1. INTRODUCTION

In Enhanced Geothermal Systems (EGS) sufficient fluid (usually water) flow between injection and production wells is critical in establishing a viable geothermal energy system and in maximising the productive life of the geothermal reservoir. In particular, the fracture pathways through which the fluid flows between the two wells should be such as to extract the maximum amount of heat over the life of the reservoir and convey it to the surface where it is used to generate electricity. Fluid transport in fracture networks (Berkowitz 2002; Karvounis and Jenny 2011) is determined by connectivity (Chiles and de-Marsily 1993; Fadakar-A et al. 2013a-2013b), topology (Huseby et al. 1997), geometrical complexity (CFCFF 1996; Fadakar-A et al. 2011) and many other characteristics of the network. For a single fracture and a simplistic model, e.g., parallel flat plate and fluid incompressibility, the fluid flow rate (Q) is obtained from Darcy's law (Priest 1993) using at least two geometrical measures of the fracture – the length (l) and the aperture (a; also b or e are used). The flow formulation simplifies to: $Q = \psi a l^{-1}$ where ψ is a coefficient accounting for fluid properties such as viscosity. Thus, the fluid flow is directly related to the aperture (i.e., positive correlation) and inversely related to the fracture lengths (i.e., negative correlation). It is worth noting that there may be many fracture pathways connecting the injection and production wells. These pathways may be simple (e.g., straight connection between the two wells via a single fracture) or combinations of multiple fractures, so-called *fracture clusters*. In either case, the lengths and apertures of the connected fractures (which we term elements) are determining parameters for fluid flow (CFCFF 1996; Sarkar et al. 2004; Fadakar-A et al. 2013b). While fractures are commonly represented for geometrical modelling purposes by line segments in two-dimensions, the modelling of fluid flow relies on the inner structure of fracture networks. In a fracture network the inner structure (so-called *backbone* or *skeleton*) is made of inter-connected fractures with dead-ends removed. Backbone is analogous to the combined representation of *nodes* and *edges* of a *graph* in *Graph theory*, indeed; see Gross and Yellen 2004; and also Priest 1993; Jing and Stephansson 2007 for further details). It effectively shows all possible pathways for fluid flow in the entire network locally or regionally, e.g., between injection and production nodes (simple example). As a result, if wells are represented by a support (a minimal sub-region of the study area, see Back 2001; Fadakar-A et al. 2012-2013b for related discussions) the evaluation of fluid flow between the wells is limited to flow through the backbone. This helps to avoid issues (such as instability in numerical computation, higher computation cost) caused by many isolated fractures or partially isolated clusters. Finally, the distance between the two wells depends primarily on the technical, topographical and design constraints. Longer distances render *surface operations* (e.g., energy extraction, fluid circulation) impractical (or infeasible) while shorter distances cannot achieve the required heat exchange (Grasby et al. 2012) in the geothermal reservoir. In the following sections we propose a method to determine the optimal locations of the injection and production wells based on a distance distribution function and optimising the fluid transport between the wells. The proposed methodology uses various scientific and engineering concepts together including fracture network connectivity, support theory, fluid transport equation, Monte Carlo sampling and graph theory.

2. FORMULATION AND METHODOLOGY

For a given fracture network a precise determination of the optimal location of the injection and production wells can be achieved by evaluating all possible locations in a study area that are separated by a distance drawn from a distribution function but still remain connected. This suggests the use of a brute-force searching (BFS) method, which is simple but also impractical. Even using an approximate BFS for a study area on a fine grid, say 500×500 square cells, the total number of pairs to be examined is $n\times(n-1)=250000\times(250000-1)$ which results in 62,499,750,000 pairs. Although the computational efficiency can be increased by various approaches, such as using pair-wise distances (which reduces the computation cost by 50%), the number of calculations required is still not practical (Fadakar-A et al. 2013c). Note also that for every pair chosen, a corresponding local backbone connecting the pair of supports must be established and then the shortest weighted pathway must be computed. The weighting factor is based on the length and aperture of the elements in the local backbone. The overall length and aperture for all elements in a local backbone is computed using an equivalent length and aperture.

2.1 Equivalent Length and Aperture for a Set of Fractures

A pathway between the two wells may have many connected fractures (examples can be found in Fadakar-A et al. 2013b). Some are parallel making the flow transport easier while some are connected in series lengthening the pathways. As the aperture and length are included in the weighting system for each pathway the equivalent aperture of connected fractures in the pathway (elements) must be calculated. For serially connected fractures the equivalent length is simply the sum of the lengths i.e., $L_{eq} = \sum_{i=1}^{n} l_i$, analogous to the serial combination of electrical resistors. For aperture, however, the relationship is analogous to combinations of resistors in parallel. The governing equations for the equivalent resistance of a set of resistors (R) are as follows, where R_{eq}^S and R_{eq}^P stand for serial and parallel connections, respectively.

$$R_{eq}^S = \sum_{i=1}^{n} R_i \qquad \text{and} \qquad R_{eq}^P = \left(\sum_{i=1}^{n} R_i^{-1} \right)^{-1} \tag{1}$$

Observations and measurements show that a set of parallel apertures acts as a set of serial resistors (Sarkar et al. 2004; Fadakar-A et al. 2013c) and thus the equivalent aperture is the sum of all apertures (Fig. 1). On the other hand, the equivalent aperture for a series of fractures (connected serially) can be obtained by means of existing proposals including (i) *electrical analogue for fractures in series* (EAS, Sarkar et al. 2004) and (ii) *porous media analogue for fractures in series* (PMAS, Bairos 2012). Moreover, the equivalent aperture in generic form can be obtained as follows.

$$a_{eq} = \left(\left(\sum_{i=1}^{n} l_i \right)^{-1} \sum_{i=1}^{n} \frac{l_i}{a_i^w} \right)^{-w} = \left(L^{-1} \sum_{i=1}^{n} l_i a_i^{-w} \right)^{-w} = L^w \left(\sum_{i=1}^{n} l_i a_i^{-w} \right)^{-w} \tag{2}$$

in which for exponent w=2 and w=3 the formula becomes EAS and PMAS, respectively.

In our application, as the ultimate aim is to determine weights for pathways rather than apertures; an easier, but still effective, solution is to analogise a fracture as a physical resistor defining $R = l/a$ (see Fig. 1 for demonstration). Accordingly, all combinations of serial and or parallel fractures are easily represented by equivalent resistors (Fig. 1). In this way, the magnitude of the pathway resistance is used as the weighting factor for the pathway between the two wells. That is, the lowest resistance suggests the most optimal corresponding pathway in the study region. Note, however, that all these methods are solely associated with the geometrical attributes of fractures and not the physical or behavioural characteristics of real fluid flow, such as gravitational force, fluid characteristics and fracture mechanics. We demonstrate (Figs. 2 to 5 in that order) the use of our proposal to establish weighting factors for the total length of pathways, however, other attributes such as the transmissivity (Zimmerman and Bodvarsson 1996; McClure 2012) can also be easily calculated; for example, using the simplistic equation of transmissivity $T = a^3\rho g/12\mu$ in which a is the hydraulic aperture assuming smooth parallel plates (L), μ is the absolute viscosity of water (M/LT), ρ is the density of water (ML^{-3}) (Haneberg et al. 1999).

To support our proposal, we observe that in an electronic resistor device, the overall resistivity (R) is related to its cross-section (A) and length (l) by a factor of special resistivity (r) which is a material property ($R = rlA^{-1}$). In the use of this concept for two-dimensional fractures, A is reduced to aperture a assuming that the third dimension is equal to 1 ($A = a \times 1 = a$). Thus the equivalent resistivity for a fracture can be effectively approximated by the ratio of length to aperture (la^{-1}). Such an analogy provides appropriate solutions for various parallel and or serial combinations of fractures in a pathway. The worked example in Fig. 1 demonstrates the use and effectiveness of our proposal for computing the weighted connectivity index (WCI, Fadakar-A et al. 2013c) for two arbitrary located wells. The proposal is adapted in the present work for weighting the pathways.

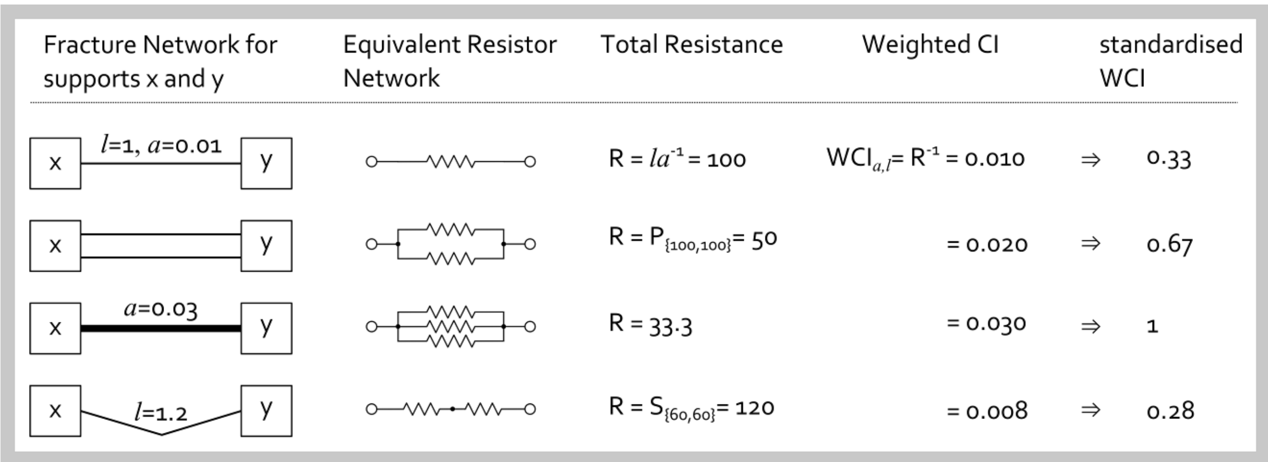

Figure 1. Equivalence between fracture and resistor networks. Worked examples of various pathway settings and equivalent weighted connectivity index. The same concept is used to determine the weight for different settings of pathways between the two wells.

2.2 Efficient Monte Carlo Sampling

An example synthetic fracture network (Fadakar-A et al. 2011) and its backbone are shown in Fig. 2 (simulation details are given in the caption). Sampling of location pairs is the next problem to be resolved. As previously stated, combinatorial explosion excludes BFS even for test cases. One solution for this problem is to use a random sampling technique such as *RANSAC* described in Fadakar-A et al. (2013c), which is an adapted form of Monte Carlo sampling. Briefly, a potential injection well location is determined randomly within the study area (Fig. 3). Any topographical or other consideration can be applied at this stage to determine the feasibility and suitability of the location and, if necessary, reject the location and generate a new one. Next, a potential production well location is drawn from a desired distance distribution function such as *Gaussian*. For any location a support with specified size $v \rightarrow 0$ is assigned. The local backbone is extracted by updating the regional backbone (Fig. 2 and 4). This is necessary because of the addition of the supports (the two wells). The shortest weighted pathway in the backbone between the two wells is determined from all possible pathways. This is done by implementing path-finder algorithms such as *dijkstra* (Gross and Yellen 2004) considering the local backbone as a sub-graph. The procedure is repeated for a maximum number n (=10,000 in Fig. 3, for example). Finally, the globally minimum value of the computed weights determines the optimal locations for the two wells (Fig. 5*left*). Furthermore, by superimposing all found shortest weighted pathways an *optimality field map* can be generated as shown in the example in Fig. 5(right) in which the darker the shade of blue the more suitable is the location with respect to the distance distribution function (e.g., $d = 0.5 \pm \alpha$, $\alpha \sim \boldsymbol{\pi}\langle N_t(0,1)\rangle_{-0.1,0.1}$ where N_t is truncated Gaussian, and $\boldsymbol{\pi}\langle . \rangle$ is projection operator). Note that in Fig. 5(*right*) only the endpoints of found pathways are used for mapping, mainly to demonstrate applicability. It can, therefore, be concluded that superimposing entire pathways (e.g., via discretisation methods) would result in more reliable mapping. The advantages of the method include: (**i**) the number of iterations is practical; (**ii**) every location in the study area has the same chance of being assessed as opposed to the grid-based method in which, regardless of the resolution, many locations are rejected. Figures 2 to 5 show the full procedure of the proposed method.

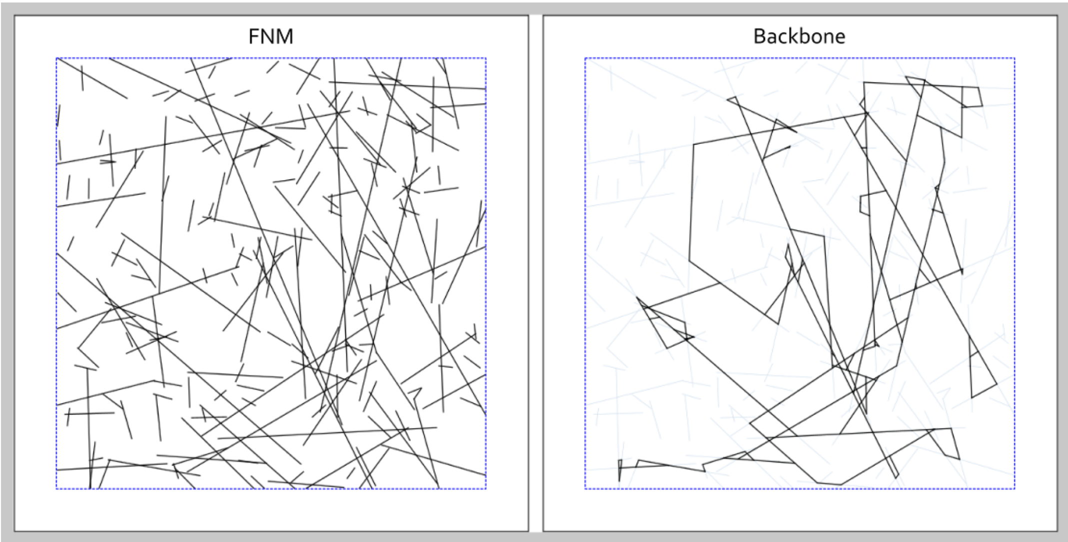

Figure 2. Synthetic fracture network ($n_{fractures}=200$, $\theta \rightarrow$ *von-Mises* $(\mu=0,\ \kappa=0)$, $\ell \rightarrow$ *Power-law* $(l_{min}=0.1,\ l_{max}=0.9)$**) and its backbone structure.**

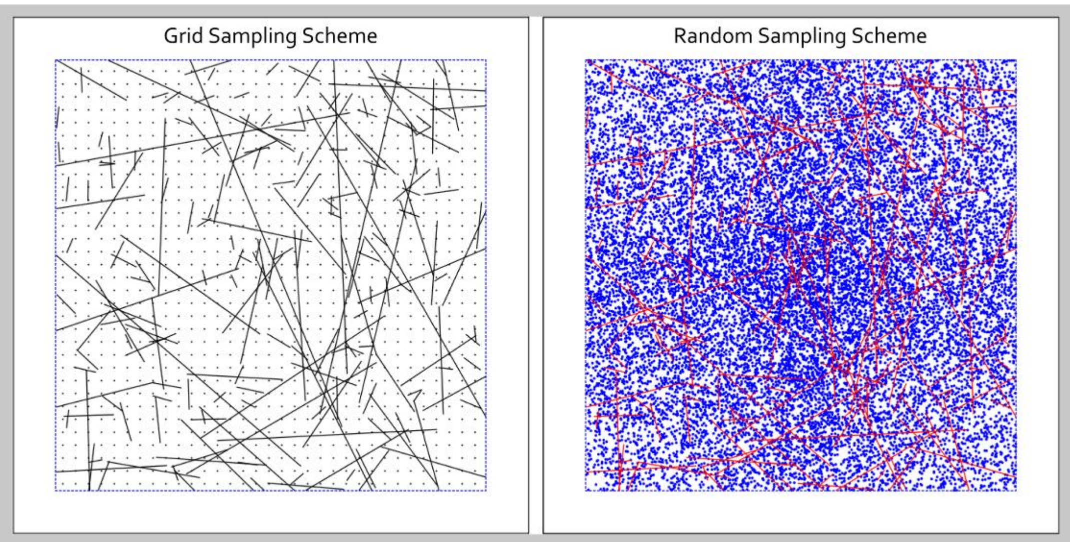

Figure 3. Grid-based (SG) and random sampling (SR) schemes. Note that in the SG scheme the locations are not pairs, that is, the distance evaluation is applied during brute-force searching; while in the SR scheme the only pairs of locations generated are those that satisfy the distance distribution function. Notice that SR gives a much higher density of sampling points while still remaining practical.

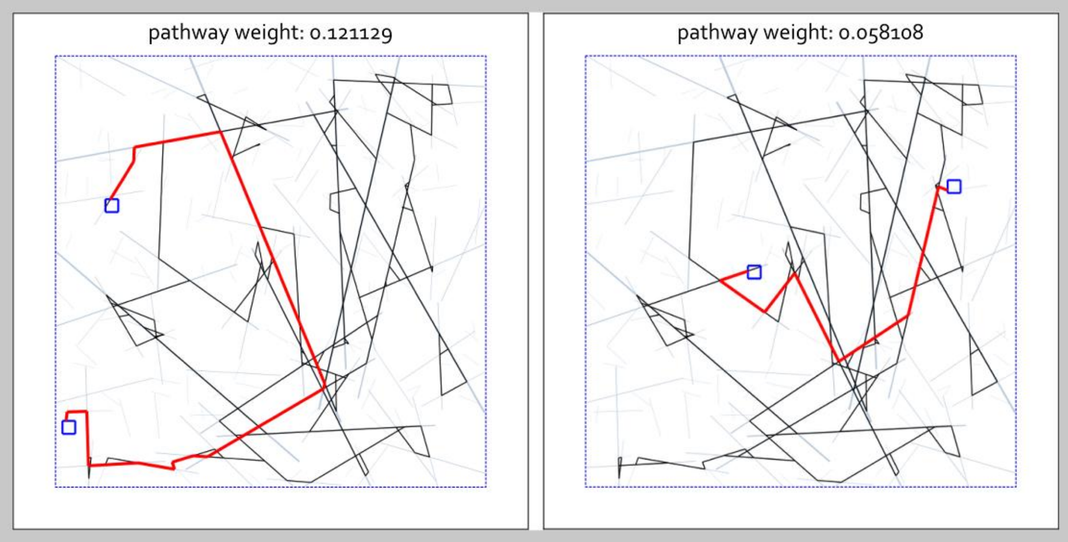

Figure 4. The shortest fluid flow pathway (red polyline; considering the effect of the length and aperture simultaneously) is determined between the two simulated wells (blue squares). The two cases are examples from a set of 10,000 trials. For each trial the total pathway weight is given at the top of the image.

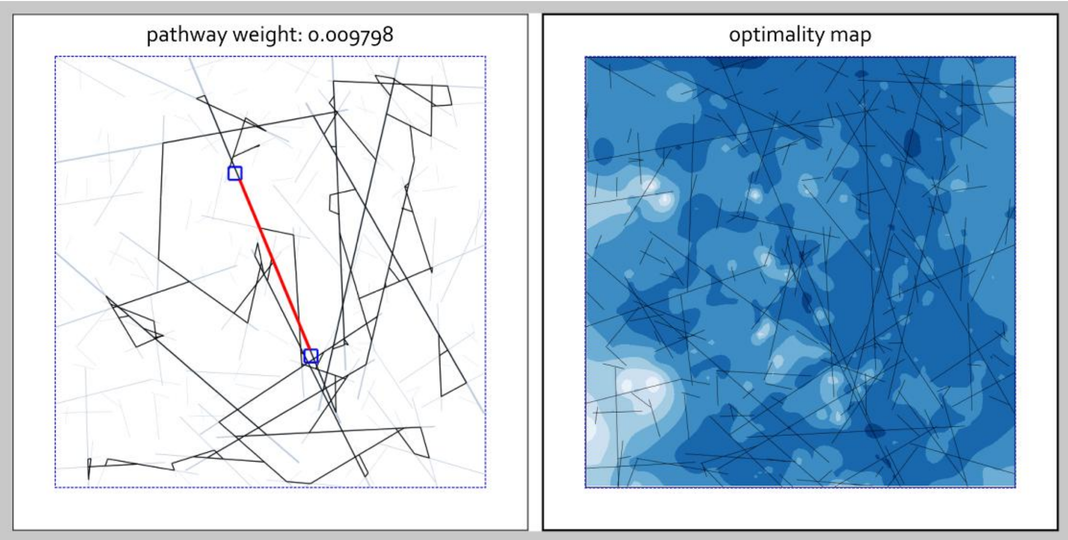

Figure 5. Outputs from the proposed procedure for optimal well locations. The pathway with the lowest weight has been determined among 10,000 trials and is shown in the *left*. The associated weight is 0.009798 while the highest weight was 0.121129. In the *right*, a regional map of the suitability of well locations is shown on which the fracture network is superimposed. The darker the blue more suitable are the locations.

3. CONCLUSION

We propose a simple but effective and realistic solution for determining optimal well locations for fracture-based reservoirs such as enhanced geothermal systems. The method is based on generating random point pairs separated by a distance drawn from a distance distribution function, for example $\mathbf{N}(\mu, \sigma)$. The parameters of the distance distribution are defined on the basis of the technical, topographical and design requirements. The proposed method uses regional and local backbone extraction for the fracture network. The pathways are determined by means of graph theory algorithms such as the dijkstra method. Each element of the local backbone is associated with a weight corresponding to its length and aperture, which in turn result in an equivalent resistivity (analogous to inappropriateness of the pathway). Note that while the length of the element is calculated locally (necessarily due to intersections and the application of backbone procedure) its aperture value is taken from the original fracture network. A number of trials are conducted using Monte Carlo line sampling for each local backbone and the shortest weighted pathways are extracted. Finally, the minimum total pathway weight suggests the optimal well locations. Our primary goal in the present work was to examine, evaluate and demonstrate the usefulness of the proposed method. The method is readily adaptable to transmissivity and other problems. The conceptually simple formulation of the method offers promise for further development and applications.

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